Instructions: Write the answers and show all your work in the bluebooks. There are 5 questions. Be sure to do all 5. You do not need to simplify answers. (I prefer final answers in the form of a fraction rather than a decimal.)

1. (7 points) Let the distribution function of a random variable $X$ be given by the following formula:

$$F(x) = \begin{cases} 
0, & x \leq 1, \\
\frac{1}{2}(x - 1), & 1 < x < 2, \\
\frac{3}{4}, & 2 \leq x < 3, \\
1, & x \geq 3
\end{cases}$$

Find

a) $P(\frac{1}{2} < X < 2)$.

b.) For what values of $a$ is $P(X = a) \neq 0$?

2. (6 points) Suppose $X$ and $Y$ are independent with the following densities respectively:

$$f_1(x) = \begin{cases} 
\frac{1}{2}x, & 0 < x < 2, \\
0, & \text{else.}
\end{cases}$$

and

$$f_2(x) = \begin{cases} 
\frac{1}{2}, & 0 < x < 2, \\
0, & \text{else.}
\end{cases}$$

Find $P(\frac{X}{2} + Y < 1)$.

3. (3 points) Let $(X,Y)$ be picked at random from the unit square. Find the probability that $Y < X^2$.

4. (11 points) Suppose the joint density function of a pair of random variables $X$ and $Y$ is given by

$$f(x, y) = \begin{cases} 
x + y, & 0 \leq x \leq 1, 0 \leq y \leq 1, \\
0, & \text{else.}
\end{cases}$$
a.) Find the marginal density of $Y$.

b.) Find $g(x|y)$, the conditional density of $X$ given $Y$, for all values of $x$ and $y$ for which it is defined.

c.) Find $P(X > \frac{1}{4}|Y = \frac{1}{2})$.

5.(6 points) Suppose the density function of $X$ is given by

$$f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty.$$  

Find the density function of $X^2$. (Hint: you must use “method 1” here.)