Instructions:

Write the answers and show all your work in the blue books. There are 5 problems. Make sure you do all 5. No books, notes, or collaboration with others.

Problem 1. (10 points) An urn contains balls numbered 1-10. 5 balls are sampled with replacement and their numbers recorded.

a. What is the probability that the sample contains five different balls?

b. What is the probability that the sample contains only even numbered balls?

c. Given that the sample contains only even numbered balls, what is the probability the balls are all different?

Problem 2. (6 points) Suppose $P(A) = 0.4$, and $P(B) = 0.1$. Find $P(A \cup B)$ and $P(A^c \cup B^c)$ if

a. $A$ and $B$ are disjoint, and

b. if $A$ and $B$ are independent.

Problem 3. (4 points) A drawer contains 6 nylon socks, 6 cotton socks, and 6 wool socks. If a sample of 4 socks is drawn at random (without replacement,) what is the probability at least one type of sock is missing from the sample?

Problem 4. (6 points) 90% of Jim’s email is spam (junk email.) He has a spam filter that rejects 75% of all spam messages (so that he never sees them), but it also rejects 10% of his legitimate email.

a. Find the probability that a given email message bound for Jim will be rejected.

b. Given that a message was rejected, what is the probability that it is spam?
Problem 5. (4 points) The Professor has a bank of 100 test questions from which he selects 10 at random for the final exam in Probability I every time he teaches the course. 50 of the questions are hard and 50 are easy. If the Professor teaches Probability I five times in his career, what is the probability that every exam will have 5 easy questions and 5 hard questions? (Assume a different set of 10 questions are selected each time, i.e., no question appears on more than one test.)