Instructions: Write the answers and show all your work in the bluebooks. There are 4 questions. Be sure to do all 4. You do not need to simplify answers.

1. (20 points) Suppose X and Y are independent with the following probability densities respectively:

\[ f_X(x) = \begin{cases} \frac{2}{x^3}, & 1 < x < 2, \\ 0, & \text{else.} \end{cases} \]

and

\[ f_Y(y) = \begin{cases} \frac{y}{2}, & 0 < y < 2, \\ 0, & \text{else.} \end{cases} \]

Find,

a) the variance of Y,

b) \( E(X) \),

c) \( E(XY) \),

d) \( E(\sqrt{X}) \), and

e) \( P(X < Y) \).

2. (5 points) Let \((X, Y)\) be the coordinates of a point picked at random from the triangle \(T\) in the plane with vertices at \((0,0)\), \((0,1)\), and \((1,0)\).

a) Write down a formula for the joint probability density function of \(X\) and \(Y\). (Hint: the joint density must be constant for \((x, y) \in T\) and zero for \((x, y) \notin T\).)

b) Are \(X\) and \(Y\) independent? Explain very briefly.

c) Find the (marginal) pdf of \(X\).

3. (5 points) If \(X\) and \(Y\) are two numbers chosen at random and without replacement from the set \{1, 2, 3\}, find the covariance of \(X\) and \(Y\). (Hint: Compute \(XY\) for each of the 3 possible samples.)
4. (5 points) The expected number of typographical errors on a page of a certain magazine is 2. What is the probability that the next page you read contains 2 or more typographical errors? Explain your reasoning. (This problem is taken directly from the course textbook.)