MAT 521          Final Exam          August 9, 2001

Instructions: Write the answers and show all your work in the bluebooks. There are 6 questions. Be sure to do all 6. You do not need to simplify answers. (A table of Normal probabilities is attached.)

Problem 1. (5 points) Joe and Jim each has 1 penny, 1 nickel, 1 dime and 1 quarter in his pocket. Each child selects one coin at random and Jim ends up with the higher-valued coin. What is the probability that Jim’s chosen coin is a dime?

Problem 2. (5 points) Jim goes to the carnival with $10 in his pocket and proceeds to play a gambling game which costs $1 each time it is played. Suppose the dollar amount won on each play is a random variable X with $P(X = 0) = \frac{3}{4}, P(X = 1) = \frac{1}{8}, P(X = 2) = P(X = 3) = \frac{1}{16}$. (The amount won does not include the $1 fee.) What is the expected value of the amount Jim will have in his pocket after he plays the game 10 times?

Problem 3. (10 points) Let X and Y have joint p.d.f given by

$$f(x, y) = \begin{cases} 2x^2, & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0 & \text{else.} \end{cases}$$

a. Find the variance of X
b. Find $P(Y < X)$.

Problem 4. (5 points) A point P is chosen at random from the interval [-1,1] on the x-axis. What is the expected value of the squared distance from P to the point (0,1) on the y-axis?

Problem 5. (5 points) A gun misfires on average about once every 2000 trigger pulls. What is the probability that the gun will misfire 2 or more times in the next 5000 trigger pulls?

Problem 6. (5 points) A farmer sets fence posts by hand, so that the actual distance between posts is random with a mean of 10 yards and a standard deviation of 2 yards. He has a supply of 103 fence posts on hand when he begins to build a fence that needs to run a distance of 1000 yards in a straight line. What is the approximate probability that he will run out of posts before the job is done? (Assume the actual distances between posts are independent.)