Instructions: Write the answers and show all your work in the bluebooks. There are 4 questions. Be sure to do all 4. You do not need to simplify answers.

1. (15 points) Suppose the joint pdf of two random variables, $X$ and $Y$, is given by

$$ f(x, y) = 4xy, \quad 0 < x < 1, 0 < y < 1. $$

a) Find the marginal pdf of $X$.

b) Calculate $P(Y < X)$.

c) Are $X$ and $Y$ independent? Carefully justify your answer.

2. (5 points) Let $X$ and $Y$ be random variables satisfying $E(X) = 0, E(Y) = 1$. Assume both variables have variance equal to one, and covariance equal to $-\frac{1}{2}$. Find $E[(X - 2Y)^2]$.

3. (5 points) Let $X$ have pdf

$$ f(x) = \frac{1}{x^2}, \quad x > 1. $$

Find the pdf of $Z = \frac{1}{X}$. (Hint: if $0 < z < 1$,

$$ P\left(\frac{1}{X} \leq z\right) = P(X \geq \frac{1}{z}) = \int_{\frac{1}{z}}^{\infty} \frac{1}{x^2} \, dx. $$

4.(5 points) A gambler is paid $5 for each head that appears in 3 coin tosses. What fee should be charged in order that this game is fair to both the gambler and the house?