Instructions:

Write the answers and show all your work in the blue books. There are 6 problems. Make sure you do all 6. No books, notes, or collaboration with others.

Problem 1. (20 points) You are given the following formula for a function $f$, and, as a courtesy, the correct formula for the second derivative of $f$:

$$f(x) = \frac{(x + 1)^2}{x^2 + 1}, \quad f''(x) = \frac{4x(x^2 - 3)}{(1 + x^2)^3}.$$ 

(a) Find all asymptotes.
(b) Determine all critical values, local extrema, and intervals on which $f$ is increasing and decreasing.
(c) Determine all inflection points and intervals where the graph of $f$ is concave up and concave down.
(d) Determine all intercepts.
(e) Sketch the graph of $f$.

Problem 2. (4 points) In order to travel a distance of $x$ light years, a starship must carry $40x^2$ tons of nuclear fuel costing 5 galactic dollars per ton. Regardless of the distance of the trip, the ship’s bursar must pay the crew a total of 200 galactic dollars. What distance trip will minimize the cost per light year? (Ignore other costs besides fuel and payroll.)

Problem 3. (4 points) Using an initial estimate of $x_0 = 0.5$, apply Newton’s method once through to get the next better estimate of the root of $3x^3 + 4x - 1 = 0$ lying between 0 and 1. Show all steps and formulas used.

Problem 4. (4 points) Suppose $f(2) = 5, f(5) = 3$, and $f$ is differentiable on $[2,5]$.

(a) Does the Mean Value Theorem apply to $f$ on the interval $[2,5]$? Briefly explain why or why not.
(b) Assuming the Mean Value Theorem is applicable, what value does it say the derivative $f'(x)$ must have at some $x$ in $(2,5)$?
Problem 5. (3 points) Find the limit shown:

\[
\lim_{x \to 1} \frac{x - 1}{x \ln x}.
\]

Problem 6. (5 points) Let \( f(x) = (1000 + x^2 - 2x)^{\frac{2}{3}} \).

(a) Find the linearization of \( f \) at \( x = 2 \).

(b) Assume that \( f \) represents the cost per item manufactured if a company manufactures \( x \) thousand items. If the company is currently producing 2000 items and is considering increasing production by 3%, by approximately what percentage can the cost of manufacturing each item be expected to change? (Do not calculate the exact percentage change. Rather, estimate it using the differential.)