Instructions:

Write the answers and show all your work in the blue books. There are 4 problems. Make sure you do all 4. No books, notes, or collaboration with others.

Problem 1. (20 points) A function $f$ is twice differentiable, satisfies $f(-x) = f(x)$, and also the following:

(i) $f'' > 0$ on $[0,1)$, and $(3,\infty)$; $f'' < 0$ on $(1,3)$.

(ii) $x = \pm 2$ and $x = 0$ are the critical points.

(iii) $f' > 0$ on $[0,2)$ and $f' < 0$ on $(2,\infty)$.

(iv) $\lim_{x \to \infty} f(x) = 0$.

(v) $f(0) = 0, f(1) = 1, f(2) = 2$ and $f(3) = 1$.

From this information, answer the following questions. Show your reasoning in each case:

(a) Determine all local maxima and minima.

(b) Determine all inflection points.

(c) Sketch the graph of $f$.

Problem 2. (8 points) Find the maximum and minimum values of

$$f(\phi) = \cos(\phi) + \sin(\phi)$$

on the interval $[0, \pi]$.

Problem 3. (6 points) Soma keeps people happy, and unhappy people cost the king money. If a man has $x$ ounces of Soma on a given day, he will cost the king $\frac{0.5}{x}$ that day. If Soma costs the king 2 cents an ounce, what amount of Soma per person per day will be most economical for the king to purchase?

Problem 4. (4 points) Suppose $f''(x) < 0$ for all $x \neq 0$. Also suppose that $f$ has a local minimum at $x = 0$. Can $f$ be differentiable everywhere? Carefully explain your reasoning.