Instructions: Write the solutions and show all your work in the bluebook. There are 5 questions. Make sure you work all 5. Each problem carries equal weight.

1. Let $A$ be a nonempty set. Consider the relation, $D$, called the diagonal relation, defined by

   $$D = \{(x, x) : x \in A\}.$$

   a. Prove that $D$ is an equivalence relation on $A$.

   b. Describe the resulting partition of $A$ into disjoint equivalence classes.

2. Let $f(x) = \frac{1}{x+3}$.

   a. What is the (implied) domain of $f$?

   b. What is the range of $f$?

   c. Show that $f$ is a bijection of the set in part (a) onto the set in part (b).

   d. Find a formula for the inverse function.

3. Let $A$ be the set of real-valued functions having domain $[0,1]$. Define a relation $\leq$ on $A$ by

   $$f \leq g \iff f(x) \leq g(x), \ 0 \leq x \leq 1.$$

   a. Prove that $\leq$ is a partial order on $A$.

   b. Show that $\leq$ is not a linear order by exhibiting a pair $f$ and $g$ that are not comparable.

4. Let $f : A \longrightarrow B$ and $g : B \longrightarrow C$ be functions. Suppose $g \circ f$ is injective. Prove that $f$ must be injective. Must $g$ be injective?

5. For images of sets we know that, in general, $f(C \cap D) \subseteq f(C) \cap f(D)$, but that equality sometimes does not hold. Prove that equality does hold if $f$ is injective.