

Answers without any explanation will receive very little credit. If you use the calculator, explain which features were used and how they were used.

**Problem 1. ( 5 points )** The *Whatsit corporation* has factories in 3 cities and the daily production (in thousands of units) of each of the 3 items they make is given in the following table:

City	Doodads	Widgets	Thingamajigs
New York	10	30	10
Philadelphia	15	5	10
Atlanta	20	20	30

It costs \$2 to make each doodad, \$1 to make each widget, and \$3 to make each thingamajig.

- a. What is the cost of a day's production in each of the three cities?

New York production =  $(10)(2) + (30)(1) + (10)(3) = 80$  (thousand dollars.) Philadelphia production =  $(15)(2) + (5)(1) + (10)(3) = 65$ . Atlanta production =  $(20)(2) + (20)(1) + (30)(3) = 150$ .

- b. Express the result of part (a) as a matrix equation using  $P$  as the name of the matrix obtained from the table.

$$P \times \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 80 \\ 65 \\ 150 \end{bmatrix}.$$

**Problem 2. ( 5 points )** Let  $A$  and  $C$  be the given matrices:

$$A = \begin{bmatrix} 1 & 4 \\ -1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}.$$

Find a matrix  $B$  such that  $A \times B = C$ .

To solve for matrix B, multiply both sides on the left by  $A^{-1}$ . Since

$$A^{-1} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$$

we have

$$B = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{4}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}.$$

**Problem 3. (5 points)** To be eligible for the lightweight division, the three men in a crew boat must average no more than 150 pounds. If Joe weighs twice as much as Jim, and Jim and Bill weigh 200 pounds together, what is the most Jim can weigh if the boat is to make weight?

Let  $x$  represent Jim's weight. Then Joe weighs  $2x$  and Bill weighs  $200 - x$ . Thus we must have  $\frac{x+2x+200-x}{3} = 150$ , or  $2x = 250$ , so  $x = 125$ .

**Problem 4. (7 points)** For what value(s) of the number  $c$  will the following system of equations have infinitely many solutions  $x$ ,  $y$ , and  $z$ ?

$$\begin{array}{rcl} x + 2y + 3z & = & c \\ -y - 2z & = & 1 \\ x + y + z & = & 5 \end{array}$$

Pivoting around the (1,1) entry of the augmented matrix we obtain

$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & c \\ 0 & -1 & -2 & 1 \\ 1 & 1 & 1 & 5 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 2 & 3 & c \\ 0 & -1 & -2 & 1 \\ 0 & -1 & -2 & 5-c \end{array} \right].$$

Similarly, pivoting around the (2,2) entry yields

$$\left[ \begin{array}{ccc|c} 1 & 0 & -1 & c+2 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 4-c \end{array} \right].$$

Thus, if  $c = 4$  there are infinitely many solutions. (For any other value of  $c$  there are no solutions.)