

Answers without any explanation will receive very little credit. If you use the calculator, explain which features were used and how they were used.

Problem 1. (5 points) John did 10 points (out of 100) better on his second test than on his first, but still needed to score at least 80% on the third and final test to pass the course. (Passing is a 60% average or better.) If the three tests counted equally, what were his scores on the first two tests?

Let x and y be the scores on the two tests. Then $y = x + 10$ and $\frac{x+x+10+80}{3} = 60$, or $2x + 90 = 180$. Thus $x = 45$ and $y = x + 10 = 55$.

Problem 2. (7 points) For what value(s) of the number c will the following system of equations have infinitely many solutions x , y , and z ?

$$\begin{aligned} x + y + z &= c \\ y + z &= 2 \\ x + 2y + z &= 5 \end{aligned}$$

Pivoting around the (1,1) entry of the augmented matrix we obtain

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & c \\ 0 & 1 & 1 & 2 \\ 1 & 2 & 2 & 5 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & c \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 5 - c \end{array} \right].$$

Similarly, pivoting around the (2,2) entry yields

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & c - 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 3 - c \end{array} \right].$$

Thus, if $c = 3$ there are infinitely many solutions. (For any other value of c there are no solutions.)

Problem 3. (5 points) The *Whatsit corporation* has factories in 3 cities and the daily production (in thousands of units) of each of the 3 items they make is given in the following table:

City	Doodads	Widgets	Thingamajigs
New York	15	20	30
Philadelphia	10	5	10
Atlanta	20	20	20

It costs \$1 to make each doodad, \$2 to make each widget, and \$3 to make each thingamajig.

- a. What is the cost of a day's production in each of the three cities?

NY production = $(15)(1) + (20)(2) + (10)(3) = 145$ (thousand dollars.)

Philadelphia production = $(10)(1) + (5)(2) + (10)(3) = 50$, and Atlanta

production = $(20)(1) + (20)(2) + (20)(3) = 120$.

- b. Express the result of part (a) as a matrix equation using P as the name of the matrix obtained from the table.

$$P \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 145 \\ 50 \\ 120 \end{bmatrix}.$$

Problem 4. (5 points) Let A and C be the given matrices:

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}.$$

Find a matrix B such that $A \times B = C$.

Solve for matrix B by multiplying both sides on the left by A^{-1} . Since

$$A^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{1}{6} \\ -\frac{1}{4} & \frac{1}{6} \end{bmatrix}$$

we obtain

$$B = \begin{bmatrix} \frac{1}{4} & \frac{1}{6} \\ -\frac{1}{4} & \frac{1}{6} \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{7}{12} & \frac{1}{3} \\ \frac{1}{12} & \frac{1}{3} \end{bmatrix}.$$