

Problem 1. (9 points) Consider the difference equation

$$y_n = \frac{2}{3}y_{n-1} + 2, \quad y_0 = 2.$$

a What value will the y_n approach for large n (i.e., what is the fixed point)?

We have $a = \frac{2}{3}$ and $b = 2$. Thus, the fixed point is $\frac{b}{1-a} = \frac{2}{1-(2/3)} = 6$.

b Is the fixed point stable (attracting), unstable (repelling), or neither? How did you know?

Stable, since $|a| = \frac{2}{3} < 1$.

c Describe the behavior of the y_n as monotone, oscillating, or neither. How did you know?

Monotone, since $a = \frac{2}{3} > 0$.

Problem 2. (5 points) Instead of investing \$5000 in his bank account (which pays 2% interest compounded twice a year,) Joey loaned the money to Bill. Bill agreed to pay the money back, plus interest, five years later. How much will Bill owe Joey when he pays him back?

$$i = \frac{.02}{2} = .01, n = 5 \times 2 = 10. \text{ So } F = 1.01^{10} \times 5000 = \$5,523.11.$$

Problem 3. (9 points) Juanita saved for her son's education by putting \$400 every three months into an account paying 4% interest compounded quarterly.

a. What will be the balance of this annuity after 20 years?

We have $i = .04/4 = 0.01$ and $n = 20 \times 4 = 80$. Thus $F = s_{80|0.01}R = (121.6715217) \times (400) = \$48,668.61$.

b. How much interest is earned in total during the 20 years?

The interest is the accumulated value less the amount paid in. Thus it is $\$48,668.61 - 80 \times \$400 = \$16,668.61$.

Problem 4. (7 points) IQ scores are normally distributed with a mean of 100 points and a standard deviation of 15 points. A **genius** is a person whose IQ score is 140 or more.

- a. What is the probability that a randomly selected person is a genius?

$$Pr(x \geq 140) = \text{normald}(140, 1000, 100, 15) = 0.0038.$$

- b. If the Carrier Dome is packed with 30,000 people, roughly how many geniuses should be present?

$30,000 \times 0.0038 = 114.9$, so we would expect about 115 geniuses in the crowd. (This assumes the crowd represents a random sample from the entire population.)