MAT 183 **Test1 Version 2** **Solutions**

Problem 1. (12 points) An urn contains 3 black balls and 1 white ball. Three balls are removed one at a time and their colors noted.

a Make a tree diagram for this experiment.

See diagram attached.

b What is the probability the last ball was black?

$(\frac{3}{4})(\frac{2}{3})(\frac{1}{2}) + (\frac{3}{4})(\frac{1}{3}) + \frac{1}{4} = \frac{3}{4}$.

c Given that the last ball was black, what is the probability the second ball removed was black?

Let A be the event the second ball is black and B the event the last is black. Then $Pr(A \cap B) = (\frac{3}{4})(\frac{2}{3})(\frac{1}{2}) + \frac{1}{4} = \frac{1}{2}$. Then $Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{1/2}{3/4} = \frac{2}{3}$.

Problem 2. (8 points) Two dice are rolled and a gambler wins $1 for each 6 that shows. (If there are no sixes, he loses his $1 entry fee.)

a. Let x be the amount won(+)/lost(-) by the gambler. Complete the probability distribution of x.

k	$Pr(x = k)$
-1	25/36
0	10/36
1	1/36

b. Find $E(x)$.

$E(x) = (-1)(\frac{25}{36}) + (0)(\frac{10}{36}) + (1)(\frac{1}{36}) = -\frac{2}{3}$.

Problem 3. (12) A college basketball team has 11 members.

a. In how many ways can the coach select a group of 5 players to start a game?

11C5 = 462.

b. In how many ways can the coach divide the team into two squads of 5 each with one left-over player?

$(11C5) \times (6C5) = 2772$.

c. The five selected players in part (a) are to be sent out onto the court one at a time for introductions. In how many ways can this be done with a particular chosen group of 5?

   5! = 120.

d. In how many ways can 5 players be chosen from the 11 and sent out for introductions?

   11P5 = 55,440.

Problem 4. (8) Let C and D be events with $Pr(C) = 0.3$ and $Pr(D) = 0.6$. Find $Pr(C \cup D)$ and $Pr(C \cap D)$ if C and D are:

a. Independent,

   We have $Pr(C \cap D) = Pr(C)Pr(D) = (0.3)(0.6) = 0.18$. Then by the inclusion-exclusion formula, $Pr(C \cup D) = Pr(C) + Pr(D) - Pr(C \cap D) = 0.3 + 0.6 - 0.18 = 0.72$.

b. Mutually exclusive.

   $Pr(C \cap D) = 0$ since the intersection is empty. By the simple union rule (applicable in the mutually exclusive case only,) $Pr(C \cup D) = Pr(C) + Pr(D) = 0.3 + 0.6 = 0.9$.