MAT 183 **Test1 Version 1** **Solutions**

Problem 1. (12) A college baseball team has twelve members.

a. In how many ways can the coach select a group of nine players to take the field? (Assume each player can play any of the 9 field positions.)

12C9 = 220.

b. How many different batting orders are possible for **each** such choice of 9 players? (i.e., in how many ways can the **selected group** be placed in order?)

9! = 362,880.

c. How many different 9 person batting orders are possible using all 12 players?

12P9 = 79,833,400

d. Redo part (a) assuming that 3 of the players can only play pitcher or first base (the rest can play any of the other 7 positions.)

By the multiplication principle we have (number of ways to choose a pitcher and first basemen)x(number of ways to choose the other 7 positions) = $(3C2) \times (9C7) = 3 \times 36 = 108$.

Problem 2. (8) Let A and B be events with $Pr(A) = 0.4$ and $Pr(B) = 0.5$. Find $Pr(A \cup B)$ and $Pr(A \cap B)$ if A and B are:

a. Mutually exclusive,

$Pr(A \cap B) = 0$ since the intersection is empty. By the simple union rule (applicable in the mutually exclusive case only,) $Pr(A \cup B) = Pr(A) + Pr(B) = 0.4 + 0.5 = 0.9$.

b. Independent.

We have $Pr(A \cap B) = Pr(A)Pr(B) = (0.4)(0.5) = 0.2$. Then by the inclusion-exclusion formula, $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7$.

Problem 3. (12 points) An urn contains 2 black balls and 1 white ball. Balls are removed one at a time and their colors noted.

a Make a tree diagram for this experiment.

See diagram attached.

b What is the probability the last ball removed was black?

$(\frac{2}{3})(\frac{1}{2})(1) + (\frac{1}{3})(1)(1) = \frac{2}{3}$.

c Given that the last ball was black, what is the probability the first ball removed was black?

Let A be the event the first ball is black and B the event the last is black. Then $Pr(A \cap B) = (\frac{2}{3})(\frac{1}{2}) = \frac{1}{3}$. Then $Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{1/3}{2/3} = \frac{1}{2}$.

Problem 4. (8 points) Three coins are tossed and a gambler wins \$1 for each head. (If there are no heads, he loses his \$1 entry fee.)

a. Let x be the amount won(+)/lost(-) by the gambler. Complete the probability distribution of x.

k	$Pr(x = k)$
-1	1/8
0	3/8
1	3/8
2	1/8

b. Find $E(x)$.

$E(x) = (-1)(\frac{1}{8}) + (0)(\frac{3}{8}) + (1)(\frac{3}{8}) + (2)(\frac{1}{8}) = \frac{1}{2}$.