

# Harvesting Lunar Eccentricity ?

Terry R. McConnell  
Syracuse University

November 7, 2014

## Abstract

A thought experiment involving elementary physics shows that it is possible, in principle, to extract a large amount of energy from the eccentricity of the moon's orbit. Could it be done in practise? Probably not.

## 1 Introduction

There is an enormous potential source of energy represented by the non-zero value of the eccentricity of the moon's orbit. Currently, that eccentricity is about 0.05, meaning that, roughly speaking, the moon's orbit is 5% "out of round". A perfectly circular orbit having the same angular momentum would be a lower energy orbit. The difference in energy amounts to approximately  $7.318 \times 10^{23}$  Joules. (See the following section for the calculation of this estimate.) For purposes of comparison, the entire world energy consumption in 2008 has been estimated [7] at approximately  $4.74 \times 10^{20}$  J.

Imagine that the earth were tethered to the moon with a band made of some very strong elastic material. (It is sufficient to imagine a rubber band.) The end of the rubber band attached to the moon could be fixed in place, since the moon's rotation period is synchronized with its period of revolution around the earth. The end attached to the earth, unfortunately, would have to slip in order to allow for the earth to rotate under it each day. This obviously would considerably complicate the engineering difficulties<sup>1</sup>, but need not deter us in a thought experiment. Let us assume the difficulties could be overcome.

Since the distance from the moon to the earth varies during each orbit due to the orbital eccentricity, the rubber band would periodically stretch and contract. The elastic forces from the rubber band exert no torque on the earth-moon system, since they are directed through its center of mass, and therefore the addition of the rubber band could not change the angular momentum of the orbit. If the band were perfectly elastic, the energy of the orbit would remain unchanged too, but frictional losses in a real (imperfectly elastic) band would gradually circularize the orbit. The heat produced by these frictional losses could conceivably be harvested for generating electricity.

Of course, constructing the quarter million mile band of rubber or any other artificial material needed to reach the moon would be a daunting engineering task, not to mention attaching the band at either end in the appropriate way. On the other hand, there has already been speculation about building tethers from the surface of the earth to satellites in geosynchronous orbits (22,000 miles.) Arthur C. Clarke [1], for example, envisioned such a tether connecting the earth to a permanent space station, built in the 22nd century. Would-be astronauts simply ride an elevator. (A stairway is provided for the fitness-minded.)

## 2 The Calculation

Since we are only interested in a rough estimate, we shall ignore the effect of the sun and other planets and assume the moon orbits the earth in a fixed elliptical orbit with eccentricity  $e$ . The total energy,  $h$ , of this

---

<sup>1</sup>Engineers on Pluto attempting to tether Charon, the major moon of Pluto, would not face this difficulty: the rotation periods of both Pluto and Charon exactly match their period of revolution.

orbit is to be compared with the energy  $h_0$  of a circular orbit having the same angular momentum,  $l$ .

Let  $a$  be the earth-moon distance at perigee, and let  $v$  be the velocity of the moon (relative to a coordinate system fixed at the center of the earth) at perigee. Since the velocity is perpendicular to the radius vector at that instant, we have  $l^2 = m^2v^2a^2$ . Here  $m$  denotes the mass of the moon, and the square of a vector quantity is to be understood as the squared magnitude. It follows that the kinetic energy of the moon at perigee is  $\frac{l^2}{2ma^2}$ . At the same instant, the potential energy of the moon is  $-\frac{mk}{a}$ , where  $k = Gm_e$ ,  $m_e$  is the mass of the earth, and  $G$  is the universal gravitational constant. Thus,

$$(2.1) \quad h = -\frac{mk}{a} + \frac{l^2}{2ma^2}.$$

Let  $a_0$  be the radius of the circular orbit with the same angular momentum,  $l$ . Then, by the same reasoning

$$(2.2) \quad h_0 = -\frac{mk}{a_0} + \frac{l^2}{2ma_0^2}.$$

To estimate  $h - h_0$ , we use a standard formula for eccentricity:

$$(2.3) \quad e = \sqrt{1 - \frac{l^2}{m^2ka}}.$$

See, for example, [5], p. 97, formula 3.62. (Comparison with [5] formula 3.49 for the inverse square force law shows that the  $k$  of formula 3.62 needs to be replaced by  $mk$  to conform with our notation.)

Replacing  $e$  with zero in (2.3) shows that  $\frac{l^2}{m^2ka_0} = 1$ , hence  $e^2 = \frac{l^2}{m^2k}(\frac{1}{a_0} - \frac{1}{a})$ . Again from (2.3),  $\frac{l^2}{m^2k} = (1 - e^2)a$ , so

$$(2.4) \quad \frac{1}{a_0} - \frac{1}{a} = \frac{e^2}{(1 - e^2)a}.$$

Subtracting (2.2) from (2.1) and using (2.4), we have

$$h - h_0 = mk \left( \frac{1}{a_0} - \frac{1}{a} \right) - \frac{l^2}{2m} \left( \frac{1}{a_0^2} - \frac{1}{a^2} \right) = mke^2 \left( \frac{1}{(1 - e^2)a} - \frac{1}{2a_0} - \frac{1}{2a} \right).$$

Since the eccentricity of the moon's orbit is small, we may approximate  $a_0$  by  $a$ , obtaining finally the approximate result

$$h - h_0 \approx \frac{mke^4}{a}.$$

The following values come from the NASA moon fact sheet [4]:

- $k = 3.986 \times 10^{14} \text{ N} - m^2/kg$
- $m = 7.342 \times 10^{22} \text{ kg}$
- $a = 3.633 \times 10^8 \text{ m}$
- $e = 0.0549$ .

Substituting these values yields the estimate of  $h - h_0$  announced in the introduction:  $7.318 \times 10^{23} J$ .

### 3 Appendix

After completing this note it occurred to us that a more practical scheme for extracting energy from the lunar orbit would be to have a tether with one end fixed at the Moon and the other end attached to a heavy object that is dragged through the earth's atmosphere. (Due to the non-zero eccentricity of the Moon's orbit, contact with the atmosphere - at least in the beginning - would have to be intermittent in order to avoid unpleasant collisions with the surface of the earth.) While in contact with the atmosphere, the heavy object would be subject to a wind of approximately 1000 miles per hour. If it were, say, a wind turbine, and the tether were made of a conducting material, energy generated by the turbine could be sent up to the Moon, where it might power a lunar civilization. Some of the energy could also be sent back to the earth by various methods.

During the extraction process the total angular momentum of the earth-moon system would be constant, but the total energy would decrease as it was used by humans, and ultimately radiated into space as waste heat. The drag on the atmosphere by the turbine would clearly work toward slowing the rotation of the earth, and reciprocally, would have the same effect on the rotation of the moon. Assuming the moon's rotation period remained tidally locked to its period of revolution around the earth, the Moon would necessarily move into higher (more distant) orbits, ones having a longer orbital period. Thus, the human energy grab would hasten the well-known recession of the moon due to its tidal interaction with the earth.

Eventually the earth-moon system would reach a state of double tidal lock, similar to the Pluto-Charon system, with the month and day equal in length, and the Moon in a geosynchronous orbit. At that point, energy extraction would grind to a halt, and the heavy object would hang uselessly in the atmosphere.

To estimate the total amount of energy that could be extracted by this method, we first need to find the radius  $a$  of a circular geosynchronous orbit for which the total angular momentum of the earth-moon system is equal to the present total angular momentum,  $l$ . We have

$$l = (I_m + I_e + ma^2)\omega$$

where  $I_m$  and  $I_e$  denote, respectively, the moments of inertia of the Moon and the Earth,  $m$  is the mass of the Moon, and  $\omega$  is the common angular velocity of the Earth and Moon's rotation, and of the Moon's revolution about the Earth. Using Kepler's 3rd Law for the common period of rotation  $\tau$ ,

$$\tau = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k}}a^{\frac{3}{2}},$$

one finds that  $a$  is the solution of the quartic equation

$$l^2 a^3 = k(I_m + I_e + ma^2)^2.$$

According to [6], the Earth's moment of inertia about its polar axis is  $I_e = 8.034 \times 10^{37} \text{ kg} - m^2$ , while, according to [4],  $I_m = 1.191 \frac{mr^2}{MR^2} I_e$ , where  $R$  and  $M$  are the radius and mass of the Earth, and  $r$  and  $m$  are the radius and mass of the Moon. Using the values  $\frac{m}{M} = 0.0123$  and  $\frac{r}{R} = 0.2727$ , also from [4], we find that  $I_m = 8.752 \times 10^{34} \text{ kg} - m^2$ .

To make the size of the numbers more manageable, we use the current value of the semi-major axis of the moon (382,868 kilometers) as distance unit, the current length of a day as the time unit, and the mass of the moon as the mass unit. With this choice of units, we have  $I_e = .007458$ ,  $I_m = 0.000008$ ,  $k = .0528869$ , and  $l = 0.2768$ . The equation satisfied by final semi-major axis of the moon's orbit is then

$$a^3 = (0.690085)(0.007467 + a^2)^2.$$

The solution is  $a = 1.4387$ . In other words, the moon's orbit will ultimately grow by about 44%. The length of the day on Earth will then equal the length of the month, measured in current days at 47.15 days.

We can easily compute the difference in energy between the current earth-moon system and its energy in the doubly tidal locked orbit. Equation (2.2) can be used to find the energy in the orbit itself. One then

also includes the rotational energy of the earth and moon by including terms  $\frac{1}{2}I_e\omega_e^2 + \frac{1}{2}I_m\omega_m^2$ , where  $\omega_e$  and  $\omega_m$  are the present day angular speeds of rotation of the Earth and Moon respectively.

The current energy in the lunar orbit turns out to be  $-0.011834$  and the total energy of the earth-moon system to be  $0.135390$ . The total energy at the time of double tidal lock is  $-0.017507$ . The energy difference of  $0.152896$  is equal to approximately  $2.2 \times 10^{29}$  Joules.

It is curious to note that, at the present time, the total energy of the earth-moon system is positive. This shows that there is enough energy in the spin of the Earth to completely eject the Moon from its orbit around the Earth, assuming all the energy could be directed to that purpose.

Once the system reaches double tidal lock, the total energy will have become negative. There is no longer enough energy to eject the Moon.

It must be recalled that mankind would be competing for energy with natural dissipation of energy by tidal heating of the earth's crust and other factors such as the effects of decreasing eccentricity of the Earth's orbit. The more quickly energy could be generated by our hypothetical wind turbine, the larger would be mankind's ultimate share of the total.

To get a rough estimate of the peak power that could be generated by a turbine, we would need to know its efficiency and cross-sectional area,  $A$ . It seems probable that a civilization capable of building a tether from the Earth to the Moon would also be capable of building a very efficient generator, so let us assume an efficiency of 80%.

As it passes, the turbine briefly accelerates the air it encounters to nearly its own speed relative to the surface of the earth, in order to spin the blades of the turbine. It leaves behind it a "wake" of air moving at the same speed, but this wake would quickly interact with the surrounding atmosphere and return to approximately a state of rest with respect to the surface of the Earth by the next time the turbine came around. (This interaction with the rest of the atmosphere is precisely the way the turbine puts the brakes on the Earth's rotation.) In the course of the day, the volume of air encountered is approximately  $2\pi RA$ . This figure must be multiplied by the mass density of air, by the square of the relative velocity, and by the efficiency in order to obtain an estimate of the energy that might be generated in one day.

Air has a density of  $1.204 \text{ kg/m}^3$  at 20 degrees Celsius and 760mm Hg. See [2], F10. According to [3], E88, the radius of the Earth is  $6378140 \text{ m}$ , so the mass of air displaced in a day by the turbine is  $4.825 \times 10^7 A \text{ kg}$ . The Earth rotates at a speed of  $1669.68 \text{ km/hr}$ , or  $463.8 \text{ m/sec}$ . This makes the kinetic energy generated in a day equal to  $5.19 \times 10^{12} A$  Joules, or  $4.15 \times 10^{12} A$  Joules at 80% efficiency. If the turbine had a radius of one mile, the area  $A$  would be approximately  $10^8$  square meters, resulting in a daily energy production that is similar to current global energy consumption in a year.

Clearly our treatment is highly simplified and there are many physical and engineering issues that have not been addressed. For example, what properties would the tether material need to have, and is it even conceivable such materials might exist? Would the tethered turbine tend to swing like a pendulum, and if so, how would this influence the practicality of the scheme? What would be the environmental impacts of dragging an object 2 miles wide through the atmosphere? Students of physics and engineering might enjoy considering these and related questions.

## References

- [1] Arthur C. Clarke, *The Fountains of Paradise*, Harcourt Brace Jovanovich, San Diego, 1979.
- [2] R.C. Weast, *CRC Handbook of Chemistry and Physics* **65**, CRC Press, Boca Raton, 1985.
- [3] The Astronomical Almanac for the Year 2000, U.S. Government Printing Office, Washington, D.C., 1998.
- [4] *Moon Fact Sheet*, <http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html>
- [5] Herbert Goldstein, *Classical Mechanics*, 2nd Ed., Addison-Wesley, Reading, MA, 1980.
- [6] K. Lambeck, *The Earth's Variable Rotation*, Cambridge University Press, Cambridge, England, 1980.

[7] [https://en.wikipedia.com/wiki/World\\_energy\\_consumption](https://en.wikipedia.com/wiki/World_energy_consumption)